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# A Stand Density Management Diagram for Balsam Fir in New Brunswick

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## **Abstract**

Balsam fir (*Abies balsamea* (L.) Mill.) is a major tree species of the Acadian Forest Region in southern New Brunswick. After harvesting operations or natural disturbances, the resulting balsam fir regeneration may require stand density management to achieve the desired future stand structure and forest products. Stand density management diagrams are a tool to assist the resource manager in examining potential results of stand density management decisions. These diagrams visually illustrate the relationships between average diameter, height, density, and volume for even-aged pure species stands. This report documents the development of a balsam fir stand density management diagram for the Fundy Model Forest.

Key Words: Acadia Forest Region, balsam fir, stand density management diagram.

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## Introduction

Stand density management diagrams (SDMDs) provide stand level summaries for making stand level silvicultural decisions. These diagrams are based on the theory of self-thinning. They show visually the relationships between average diameter, height, density, and volume for even-aged pure species stands (Archibald and Bowling 1995).

The objective of this project was to produce a stand density management diagram for balsam fir, appropriate for use in the Acadian Forest Region of southern New Brunswick. A secondary objective was to verify the use of balsam fir site index curves that were developed by Ker and Bowling (1991) for the data set.

## Data

The data consisted of untreated (control) plots and late thinning experiments from the Acadia Research Forest, pre-commercially thinned stands from southern New Brunswick (J.D. Irving Ltd. and Southern New Brunswick Wood Co-operative Ltd.), and the balsam fir dominated plots from the New Brunswick provincial PSP program. The data were compiled to a standard format with a few exceptions. If raw data were available, height-diameter (dbh) curves were estimated for balsam fir by plot and measurement and by dataset for the other species. The curve form used for the height- diameter (dbh) curves was the Weibull function.

$$ht = 1.3 + \beta_0 \cdot (1 - e^{-\beta_1 \cdot dbh^{\beta_2}})$$

For some of the younger stands, height was available for all trees but diameter at breast height (dbh) was not available for the smaller trees (generally trees shorter than 3m). For these trees, dbh was estimated as a linear function of height.

$$dbh = \beta_1 \cdot ht$$

The plot data were then compiled to the stand level. Top height was defined as the average height of the 100/plot\_area number of trees of largest dbh (e.g., for a plot area of 0.04ha, it is the average of the 4 thickest trees). The volumes were estimated using equation 14 for total volume from Honer *et al.* (1983). Only plots with 80% or more of the basal area in balsam fir were used for the development of the balsam fir SDMD. The data used for the development of the balsam fir SDMD are summarized in Table 1.

Study M-206 from the Acadia Research Forest consists of balsam fir thinned at about age 35. These stands appeared to have different height-density-volume relationships than the rest of the data and were omitted from the analyses.

**Table 1.** A summary of the data sets used.

Attribute	All data excluding M-206	
	Sample size	Mean (range)
Density (stems/ha)	174	3364 (525-23712)
Top height (m)	174	15.0 (5.2 – 18.8)
Basal area (m <sup>2</sup> /ha)	174	35.2 (2.7 – 62.5)
Volume (m <sup>3</sup> /ha)	174	205.3 (5.0 – 390.6)
Quadratic dbh (cm)	174	14.2 (4.0 – 22.2)

## Analysis and Results

### ***Maximum Size Density Line***

Only the unthinned plots were used for the maximum size density line. That is, all plots from M-301, plot 9 from M-212, and the New Brunswick PSPs. Since all plots showed evidence of mortality between measurements (Figure 1), all measurements were used for the development of the maximum size density line. A principal component line was fit through the data. A principal component line minimizes the sum of the squared perpendicular distances between the observations and the line. It has the property that the principal component line of Y on X is the same as the principal component line of X on Y. It is used when the same equation is used to predict Y from X or X from Y. It is used here because sometimes there is interest in estimating the maximum mean tree volume for a given density and sometimes there is interest in estimating the density corresponding to a given mean tree volume. The following equation was fit using principal components.

$$(1) \quad \ln(\text{volume}) = \beta_0 + \beta_1 \cdot \ln(\text{density})$$

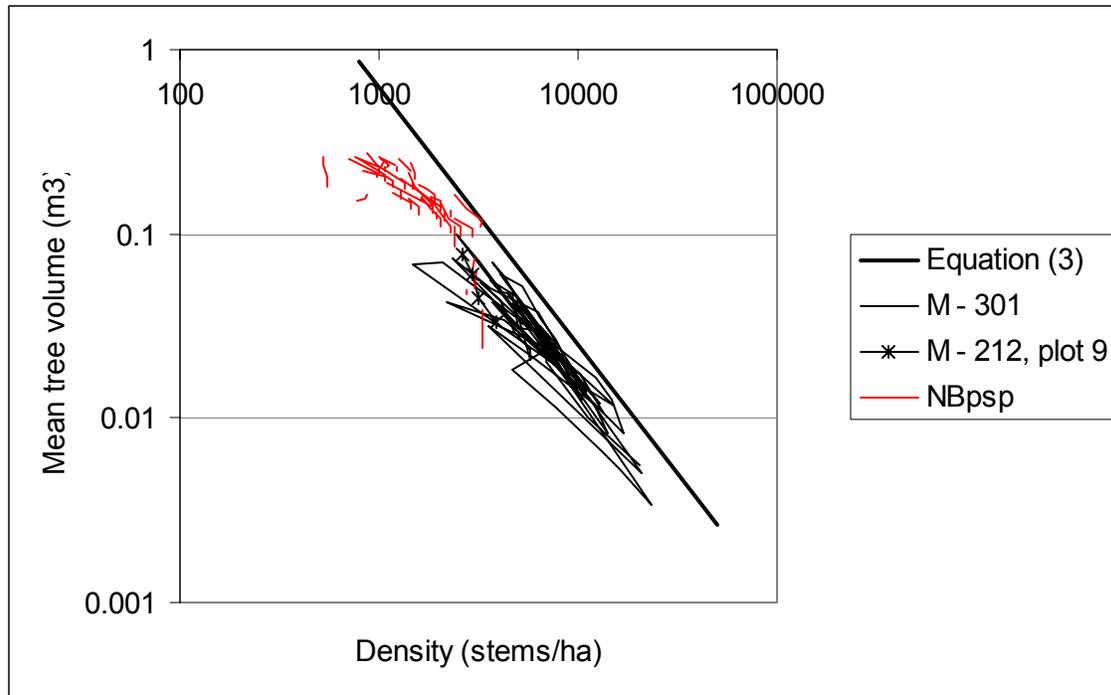
The following equation resulted.

$$(2) \quad \ln(\text{volume}) = 8.44052 - 1.39640 \cdot \ln(\text{density})$$

The equation was based on 157 observations and the first principal component accounted for 97% of the variation.

The slope of the line was held constant and the intercept increased until all the observations were on or below the maximum size density line. This resulted in the following equation.

$$(3) \quad \ln(\text{volume}) = 9.17613 - 1.39640 \cdot \ln(\text{density})$$



**Figure 1.** The data used for fitting the maximum size density line are plotted along with equation (3).

For comparison, the Smith and Woods (1997) approach for fitting maximum size density line was tried. The natural logarithm of density was put in classes 0.1 units wide. Within each log-density class, the observation with the largest mean tree volume was used to estimate the maximum size density line. Equation (1) was fit using principal components, resulting in the following equation.

$$(4) \quad \ln(\text{volume}) = 8.61993 - 1.39832 \cdot \ln(\text{density})$$

This is very similar to the equation fit using all the control plots. Equation (3) was used in the balsam fir stand density management diagram.

### ***Diameter at Breast Height (Dbh) Isolines***

The quadratic mean diameter ( $dbh_q$ ) is the diameter at breast height (dbh) of the tree of the mean basal area. To fit the  $Dbh_q$  isolines, the pre-commercially thinned plots were added. When plotted, the relationship between  $\ln(\text{mean tree volume})$  and  $\ln(\text{density})$  looked linear for a given  $dbh_q$  and parallel as  $dbh_q$  changed. In effect, the  $dbh_q$  acts as a scalar, increasing the intercept of the relationship between  $\ln(\text{volume})$  and  $\ln(\text{density})$ . Therefore, the following equation was fit for the dbh isolines using least squares regression.

$$(5) \quad \ln(\text{volume}) = \beta_0 + \beta_1 \cdot \ln(dbh_q) + \beta_2 \cdot \ln(\text{density})$$

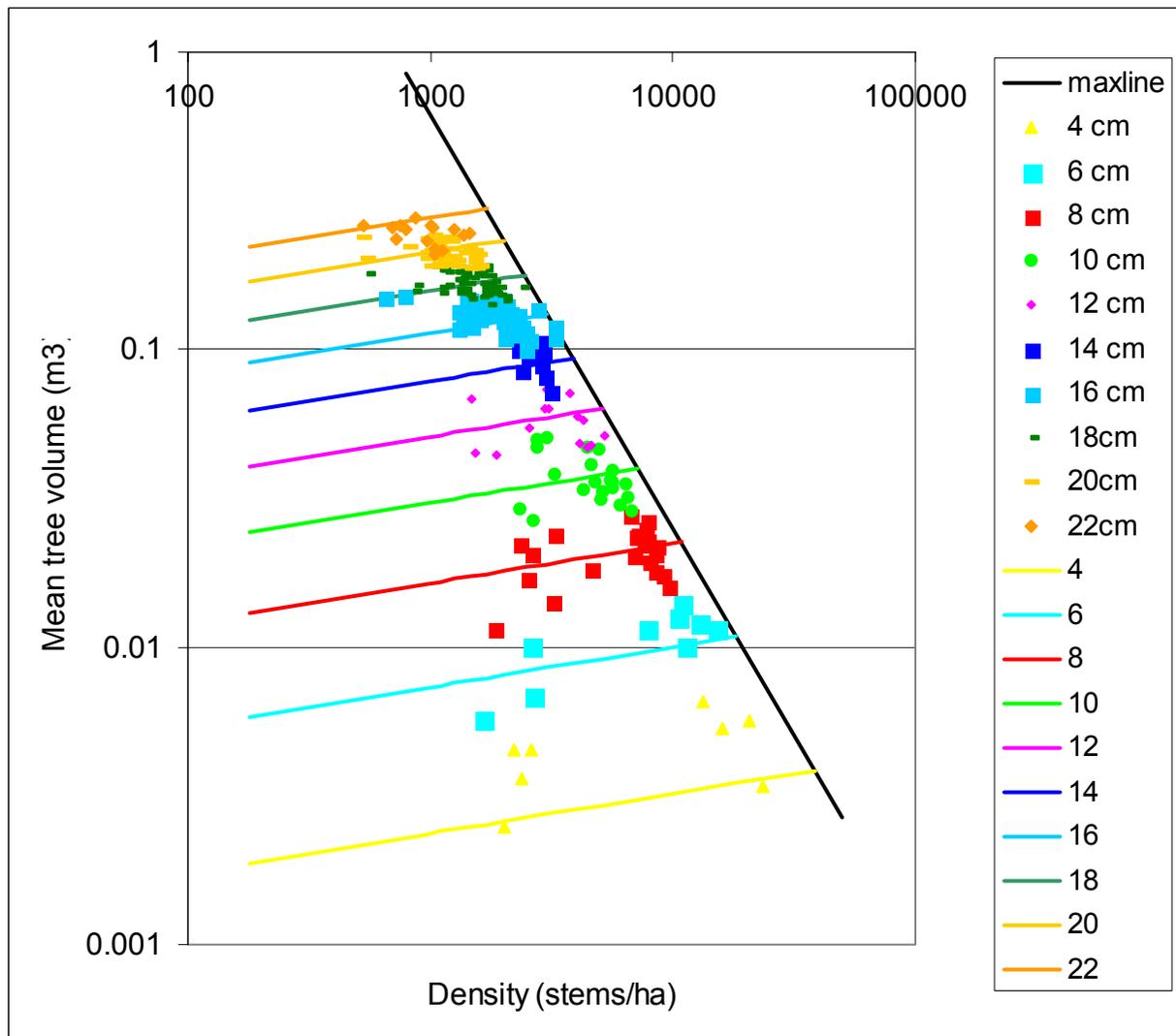
The term  $\beta_0 + \beta_1 \cdot \ln(dbh_q)$  acts as the intercept between  $\ln(\text{volume})$  and  $\ln(\text{density})$ . The following equation resulted

$$(6) \quad \ln(\text{volume}) = -10.90316 + 2.81582 \cdot \ln(\text{dbh}_q) + 0.13455 \cdot \ln(\text{density})$$

The equation was based on 174 observations and resulted in a mean squared error of 0.00607 and an  $r^2$  of 0.99. The average prediction error based on the untransformed volume was  $0.00633 \text{ m}^3/\text{tree}$  calculated as follows.

$$(7) \quad \sqrt{\frac{\sum (\text{actual volume} - \text{predicted volume})^2}{n}}$$

The data are plotted in Figure 2.



**Figure 2.** The maximum size density line and the dbh isolines are plotted along with the actual data.

### Top Height Isolines

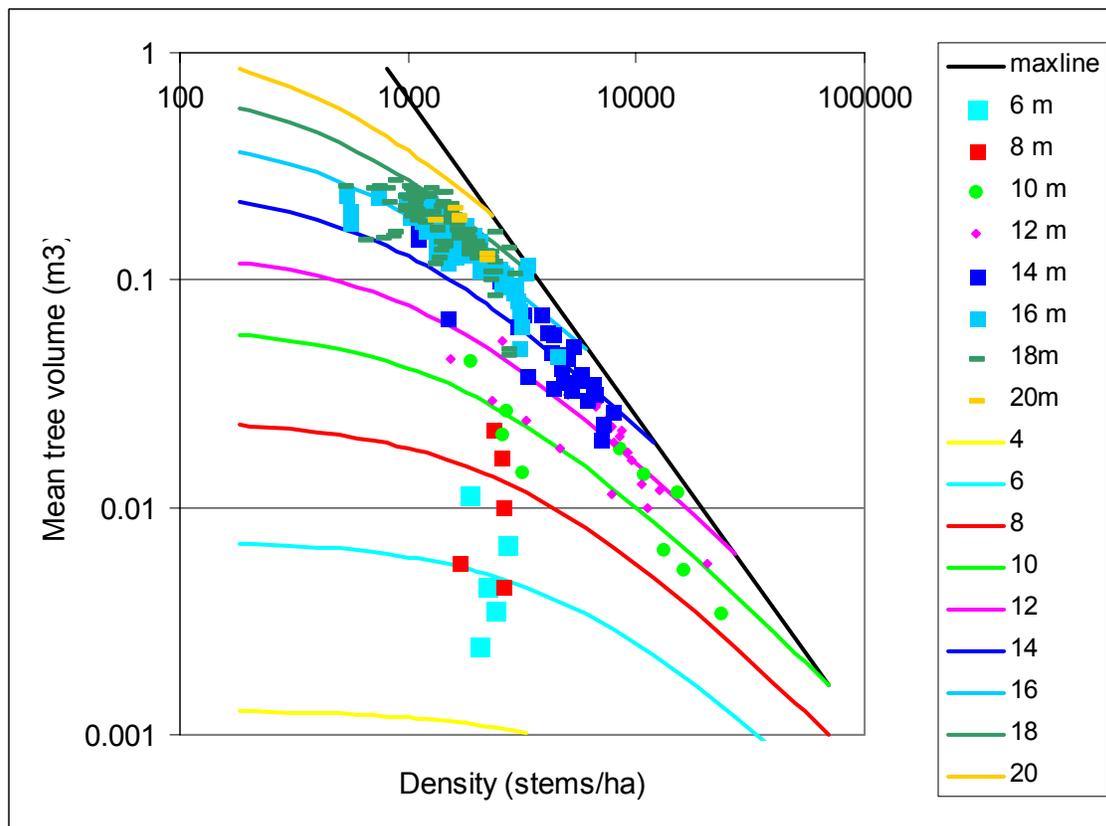
The data used to fit the Dbh isolines were also used to fit the top height isolines. The relationship between  $\ln(\text{volume})$  and  $\ln(\text{density})$  was not parallel between height classes. The height appeared to affect the slope of the relationship. The following equation was fit for the top height isolines using non-linear least squares regression. This relationship curves downward as density increases and the slope of the relationship between  $\ln(\text{volume})$  and  $\ln(\text{density})$  increases as top height increases.

$$(8) \quad \frac{1}{\text{volume}} = \beta_0 \cdot \text{topht}^{\beta_1} + \beta_2 \cdot \text{density} \cdot \text{topht}^{\beta_3}$$

Observations with smaller mean tree volumes get relatively larger weights than observation with larger mean tree volumes. To ensure the equation fit the observations with larger mean tree volumes, the mean tree volume was used to weight the residuals. The following equation resulted.

$$(9) \quad \frac{1}{\text{volume}} = 278876 \cdot \text{topht}^{-4.2426} + 1.2691 \cdot \text{density} \cdot \text{topht}^{-2.1791}$$

The equation was based on 174 observations and resulted in a mean squared error of 2.1056 and an  $r^2$  of 0.93. If calculated in the original units ( $\text{m}^3/\text{tree}$  rather than  $\text{tree}/\text{m}^3$ ), using equation (7), the square root of the average squared prediction error is  $0.03667 \text{m}^3/\text{tree}$ .



**Figure 3.** The height isolines from equation (9) are plotted along with the data.

The relationship between top height, density, and mean tree volume is not strong. For instance, there is a great deal of overlap between the 10 and 14m top height classes in Figure 2. This made it difficult to model. Several other variations for the height isoline equation were also fit.

$$(10) \quad volume = \frac{1}{\beta_0 \cdot topht^{\beta_1} + \beta_2 \cdot density \cdot topht^{\beta_3}}$$

This equation fit the data poorly. Another equation form was fit.

$$(11) \quad \ln(volume) = \beta_0 + \beta_1 \cdot \ln(dbh_q) + \beta_2 \cdot \ln(topht)$$

In untransformed form, the equation is the following, a fairly common individual tree volume form (but note that stand level parameters are used here).

$$(12) \quad volume = e^{\beta_0} \cdot dbh_q^{\beta_1} \cdot topht^{\beta_2}$$

However, the straight line relationship between  $\ln(volume)$  and  $\ln(density)$  for a given height did not fit the data well for either the untransformed or transformed equation. Likewise, the height isoline equation of Smith (1989) given below did not fit the data well because of its linear form.

$$(13) \quad \ln(topht) = \beta_0 + \beta_1 \cdot \ln(volume) + \beta_2 \cdot \ln(density)$$

This can be rewritten as the following.

$$(14) \quad \ln(volume) = \beta'_0 + \beta'_1 \cdot \ln(topht) + \beta'_2 \cdot \ln(density)$$

### ***The Balsam Fir Stand Density Management Diagram***

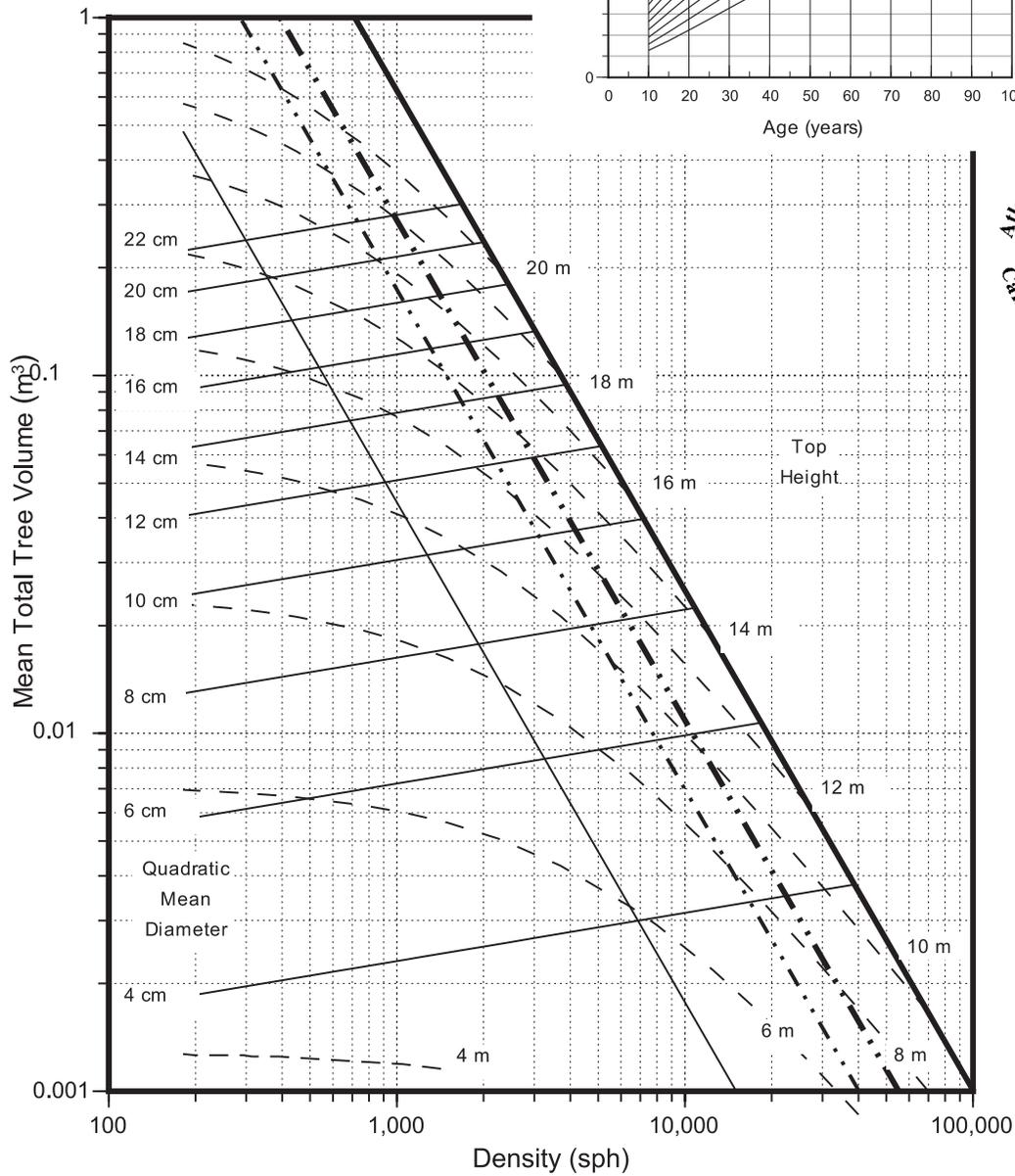
Equations (3), (6) and (9) were used to derive the DMD for balsam fir given in figures 4 and 5. In addition, relative density lines (RDI lines) were added, parallel to the maximum size density lines. The lines correspond to RDIs of 0.15 (approximate crown closure), 0.40 (lower bound for the zone of optimum growth), and 0.55 (upper bound for zone of optimum growth, and the beginning of competition-related mortality). These lines were taken from Smith and Woods (1997) and not derived from the data. The site index curves were obtained from Ker and Bowling (1991).

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Balsam Fir  
Density Management Diagram  
for Southern NB (log/log scale).  
Draft 2001

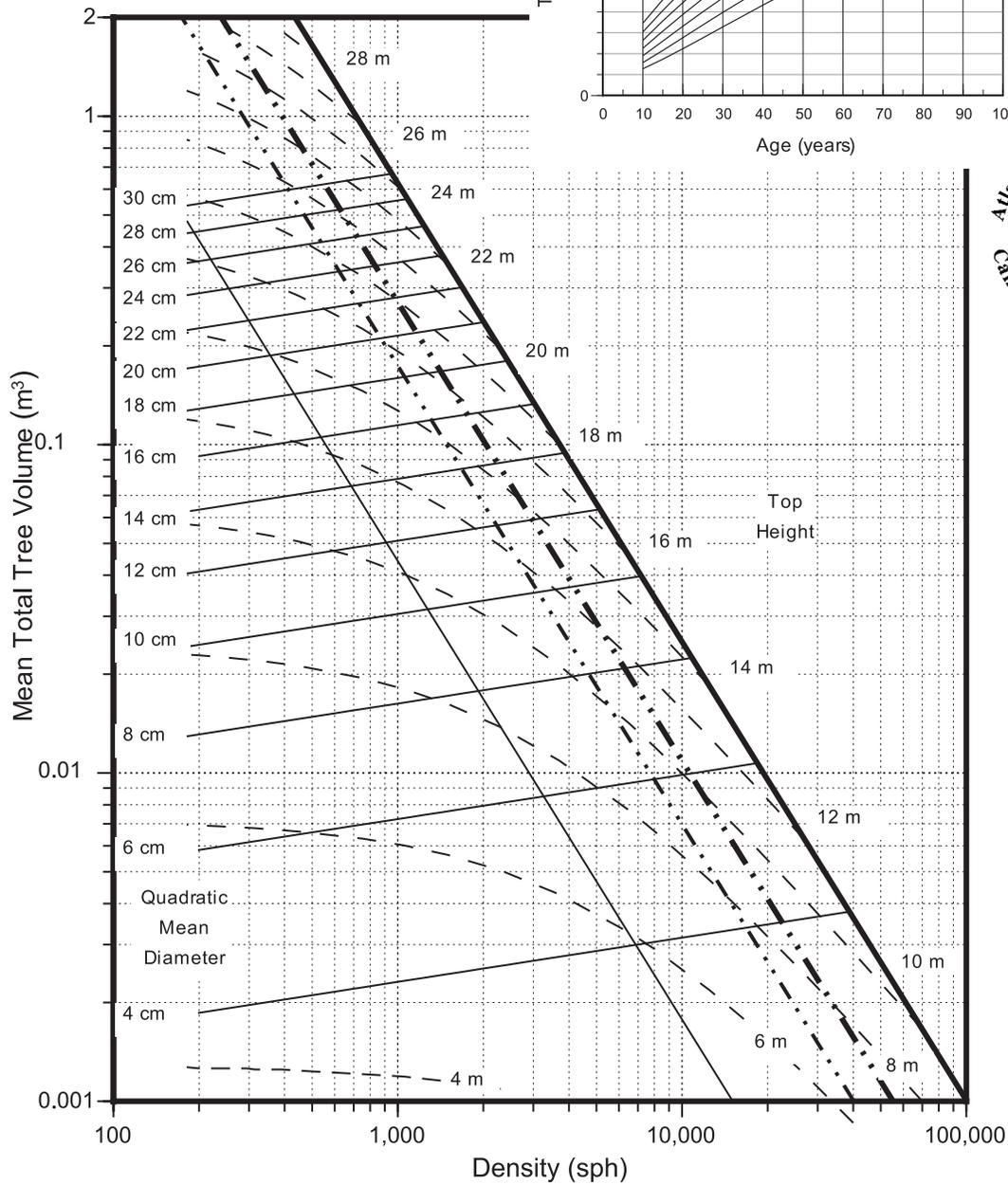
Site index curves (Ker and Bowling 1991)



**Figure 4.** The DMD for balsam fir in the Acadian region of southern New Brunswick is based on equations (3), (6), and (9).

Balsam Fir  
Density Management Diagram  
for Southern NB (log/log scale).  
Draft 2001

Extended Version



Site index curves (Ker and Bowling 1991)

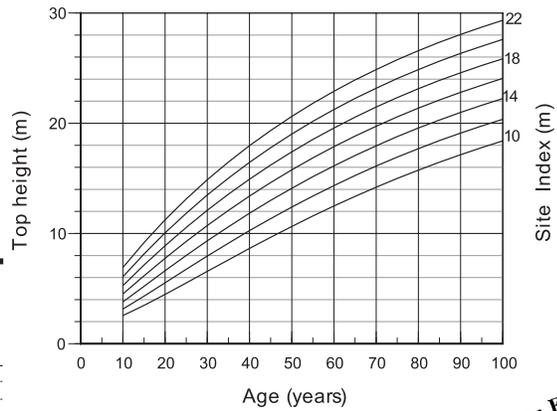


Figure 5. The same DMD as in Figure 4 but with additional height and dbh isolines.